Efficient Preparation of Cyclic Quantum States

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Outline

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Quantum States

- 1-qubit states

$$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix}\alpha\\ \beta\end{bmatrix}$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \qquad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

- n-qubit states

$$|\varphi\rangle = \sum_{i=0}^{2^n-1} \alpha_i|i\rangle = \begin{bmatrix}\alpha_0\\ \alpha_1\\ \vdots\\ \alpha_{2^n-1}\end{bmatrix}$$

$$\sum_{i=0}^{2^n-1} |\alpha_i|^2 = 1$$

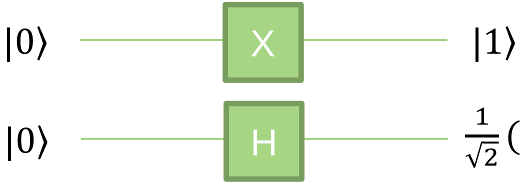
Elementary Quantum Gates

- **1-qubit gates**

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

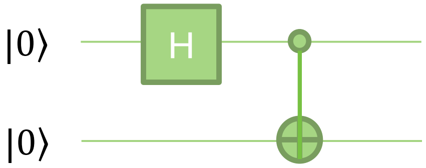
$$\mathrm{Ry}(\theta) = \begin{bmatrix} cos\frac{\theta}{2} & -sin\frac{\theta}{2} \\ sin\frac{\theta}{2} & cos\frac{\theta}{2} \end{bmatrix}$$



$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

- **2-qubit gates**

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Quantum State Preparation

- Every quantum algorithm requires a special initial quantum state
- Quantum state preparation (QSP) is an important task in quantum computing
- In general, QSP requires $O(2^n)$ elementary quantum gates and time

Approximate preparation

Using ancilla qubits

Restricting to special family of states

Dicke State

- Equal superposition of all n-qubit states $|x\rangle$ with Hamming weight wt(x) = k

$$|D_k^n\rangle = \frac{1}{\sqrt{\binom{n}{k}}} \sum_{x \in \{0,1\}^n, wt(x)=k} |x\rangle$$

- Exp. $|D_2^4\rangle = \frac{1}{\sqrt{6}} (|1100\rangle + |0110\rangle + |0011\rangle + |1001\rangle + |0101\rangle + |1010\rangle)$
- Usage: quantum sensors, quantum networking, quantum game theory, quantum metrology, combinatorial optimization problems
- State-of-the-art [1] prepares them using O(kn) gates and O(n) depth

[1] Bärtschi, Andreas, and Stephan Eidenbenz. "Deterministic preparation of dicke states." *International Symposium on Fundamentals of Computation Theory*. Springer, Cham, 2019.

Cyclic States

- Equal superposition of all n-qubit states $|1\rangle^{\otimes k}\,|0\rangle^{\otimes m}$ (m = n-k) by performing the group of cyclic permutations C(n)

$$|C_k^n\rangle = \frac{1}{\sqrt{n}} \sum_{\pi \in C(n)} \pi\left(|1\rangle^{\otimes k}\,|0\rangle^{\otimes m}\right)$$

- Exp. $|C_2^4\rangle = \frac{1}{\sqrt{4}}\left(|1100\rangle + |0110\rangle + |0011\rangle + |1001\rangle\right)$
- Uniform superposition of basis states with equal hamming weights and adjacent ones

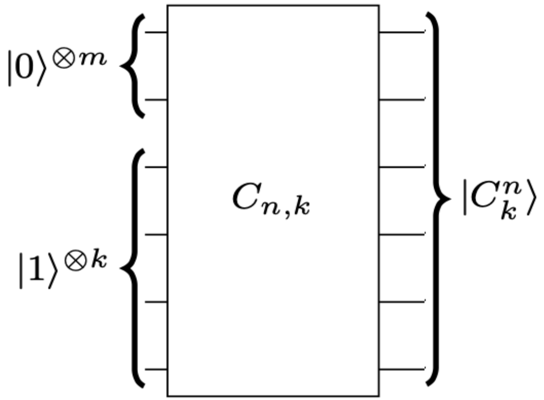
Motivation

- Cyclic states are more versatile than Dicke states in network applications
- Network applications require partial symmetry rather than fully symmetry
- Cyclic states have widespread attention for tasks in quantum internet [1] and quantum metrology [2]

[1] H. J. Kimble, “The quantum internet,” Nature, vol. 453, no. 7198, pp. 1023–1030, 2008.
[2] V.Giovannetti,S.Lloyd,andL.Maccone,“Quantummetrology,” Physical Review Letters, vol. 96, no. 1, p. 010401, 2006.

Problem Definition

- Input: Cyclic state represented by #qubits = n and #ones = k
  - #Zeros: m = n-k
  - We assume k>=m, otherwise we need to add NOT gates in the begining
- Output: Quantum circuit that load the desired Cyclic state

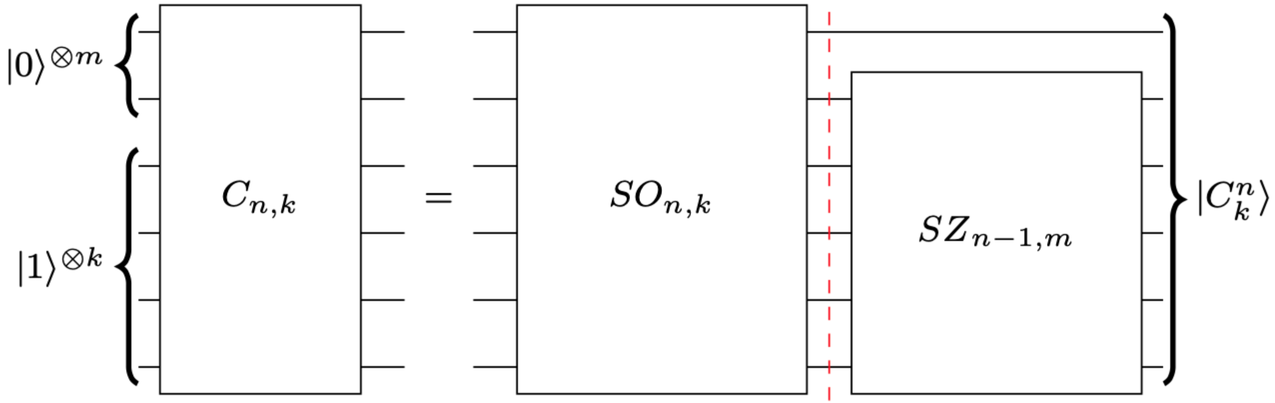


Proposed Method

- Initial state: $|1\ldots10\ldots0\rangle$
- Permutations: $|1\ldots10\ldots0\rangle, |01\ldots10\ldots0\rangle, \ldots, |0\ldots01\ldots1\rangle,$ $|10\ldots01\ldots1\rangle, |110\ldots01\ldots1\rangle, \ldots, |1..10\ldots01\rangle$

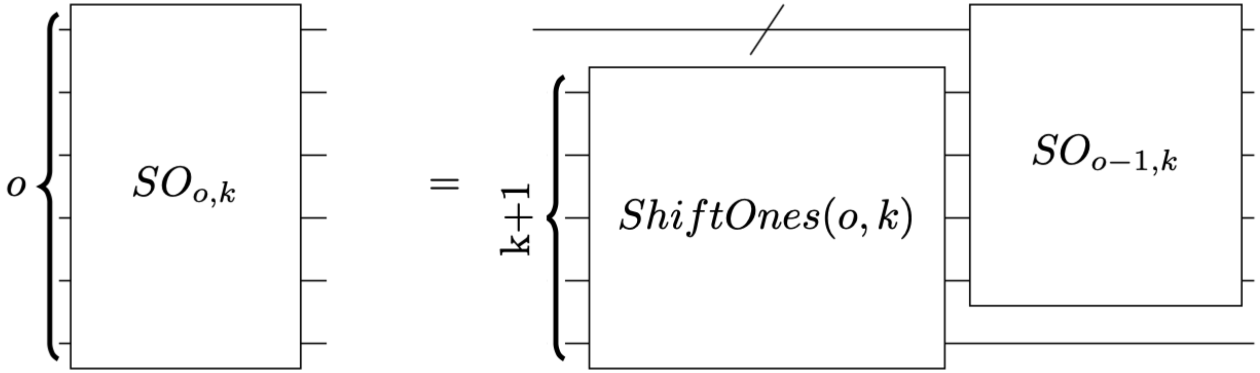
Shift ones on n qubits

Shift zeros on n-1 qubits



Construction of SO Block

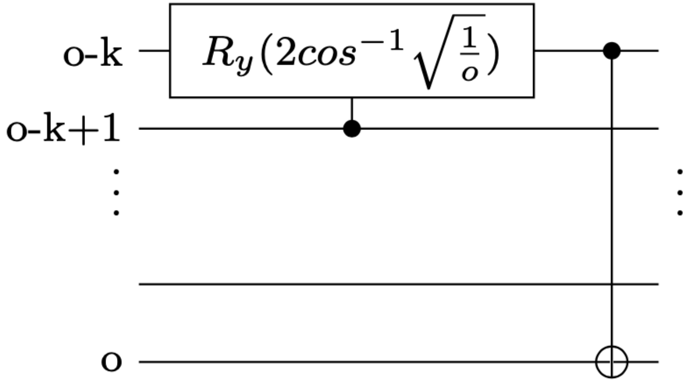
- Apply ShiftOnes unitary on k+1 qubits
- Apply SO block recursively on o-1 qubits



ShiftOnes Unitary Implementation

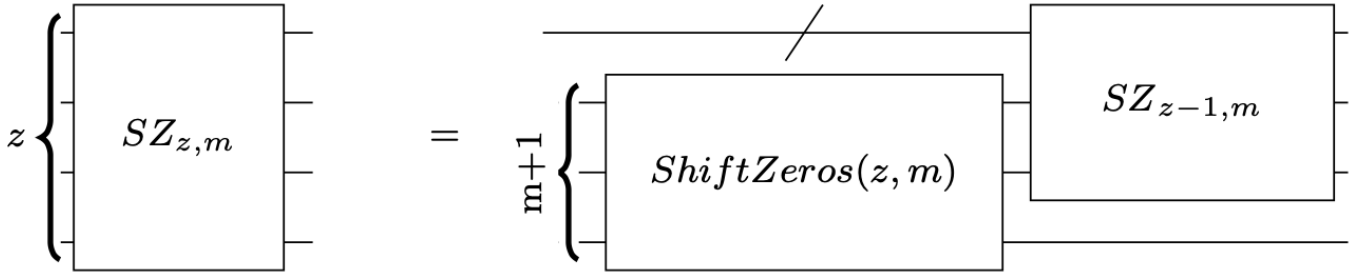
- We need to transform $|1\rangle^{\otimes k}|0\rangle \rightarrow |0\rangle|1\rangle^{\otimes k}$
- We design a quantum circuit that maps

$$|1\rangle^{\otimes k}|0\rangle \rightarrow \sqrt{\tfrac{o-1}{o}}|0\rangle|1\rangle^{\otimes k} + \sqrt{\tfrac{1}{o}}|1\rangle^{\otimes k}|0\rangle$$



Construction of SZ Block

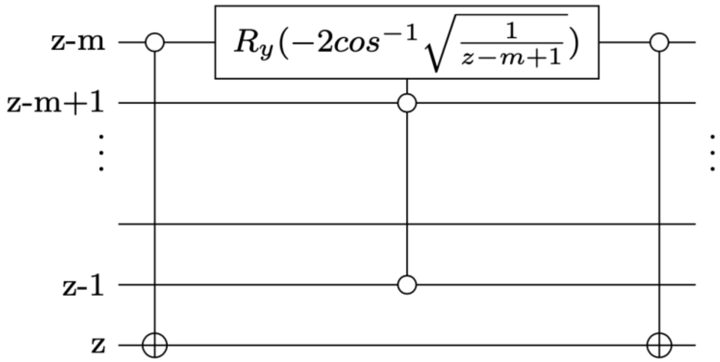
- Apply ShiftZeros unitary on m+1 qubits
- Apply Sz block recursively on z-1 qubits



ShiftZeros Unitary Implementation

- We need to transform $|0\rangle^{\otimes m}|1\rangle \rightarrow |1\rangle|0\rangle^{\otimes m}$
- We design a quantum circuit that maps

$$|0\rangle^{\otimes m}|1\rangle \rightarrow \sqrt{\frac{z-m}{z-m+1}}|1\rangle|0\rangle^{\otimes m} + \sqrt{\frac{1}{z-m+1}}|0\rangle^{\otimes m}|1\rangle$$



Complexity

- We compute the number of elementary quantum gates {CNOT, Ry, NOT}

- SO block: m ShiftOnes
- SZ block: k-1 ShiftZeros

	1-controlled Ry	2-controlled Ry	CNOT
SO block	m	0	m
SZ block	0	K-1	2k-2

- #CNOT: in the worst case → 6n-15
- Our circuit construction require O(n) elementary gates

Evaluation (1)

- We compare our approach with state of the art in term of the number of CNOTs (CNOTs are more expensive than 1–qubit gates in NISQ)
- [1] and [2] only prepare cyclic states with k=1, 2, n-1
- Our method works for arbitrary values of k

	#CNOT	#Ry
[1] $k = 2$	$\frac{7}{2}n + 3$	$4n + 2$
Proposed Method	$4n - 6$	$2n - 2$
[2] $k = 1$	$> 5n$	$> 4n$
Proposed Method	$4n - 5$	$2n - 2$
[2] $k = n - 1$	$> 5n$	$> 4n$
Proposed Method	$4n - 5$	$2n - 2$

[1] A.Burchardt,J.Czartowski,andK.Z ̇yczkowski,“Entanglement in highly symmetric multipartite quantum states,” arXiv preprint arXiv:2105.12721, 2021.
[2] A. B ̈artschi and S. Eidenbenz, “Deterministic preparation of dicke states,” in International Symposium on Fundamentals of Computation Theory. Springer, 2019, pp. 126–139.

Evaluation (2)

- We compare our approach in term of the number of CNOTs with [1] which can prepare cyclic states with arbitrary values of k
- When the number of qubits (n) is large, we reduce CNOTs with linear complexity, for all values of k .

n	10				12				15				17				19			
k	1	2	5	7	1	4	7	10	1	2	5	7	1	2	5	8	1	2	7	9
[1]	519	107	35	72	2057	262	86	300	16396	1089	1034	282	65550	2161	570	542	262160	8304	1058	1058
Proposed Method	35	34	39	45	43	42	51	42	55	54	54	60	63	62	60	69	71	70	72	78
Improvement (%)	93.3	68.2	-11.4	37.5	97.9	84.1	40.7	86.0	99.7	95.0	94.8	78.7	99.9	97.1	89.4	87.3	99.9	99.1	93.2	92.6

[1] F. Mozafari, H. Riener, M. Soeken, and G. De Micheli, “Efficient boolean methods for preparing uniform quantum states,” in IEEE Transactions on Quantum Engineering (TQE), *2*, pp.1-12, 2021.

Conclusion

- Efficient quantum state preparation is a crucial step to design quantum computing systems.
- We restrict input space to an important family of quantum states, called cyclic state.
- We propose an efficient algorithm to prepare cyclic states
- Our method requires linear circuit size while state of the art creates a circuit in exponential size.

Thanks!

Any questions?

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