# Limiting the Search Space in Optimal Quantum Circuit Mapping

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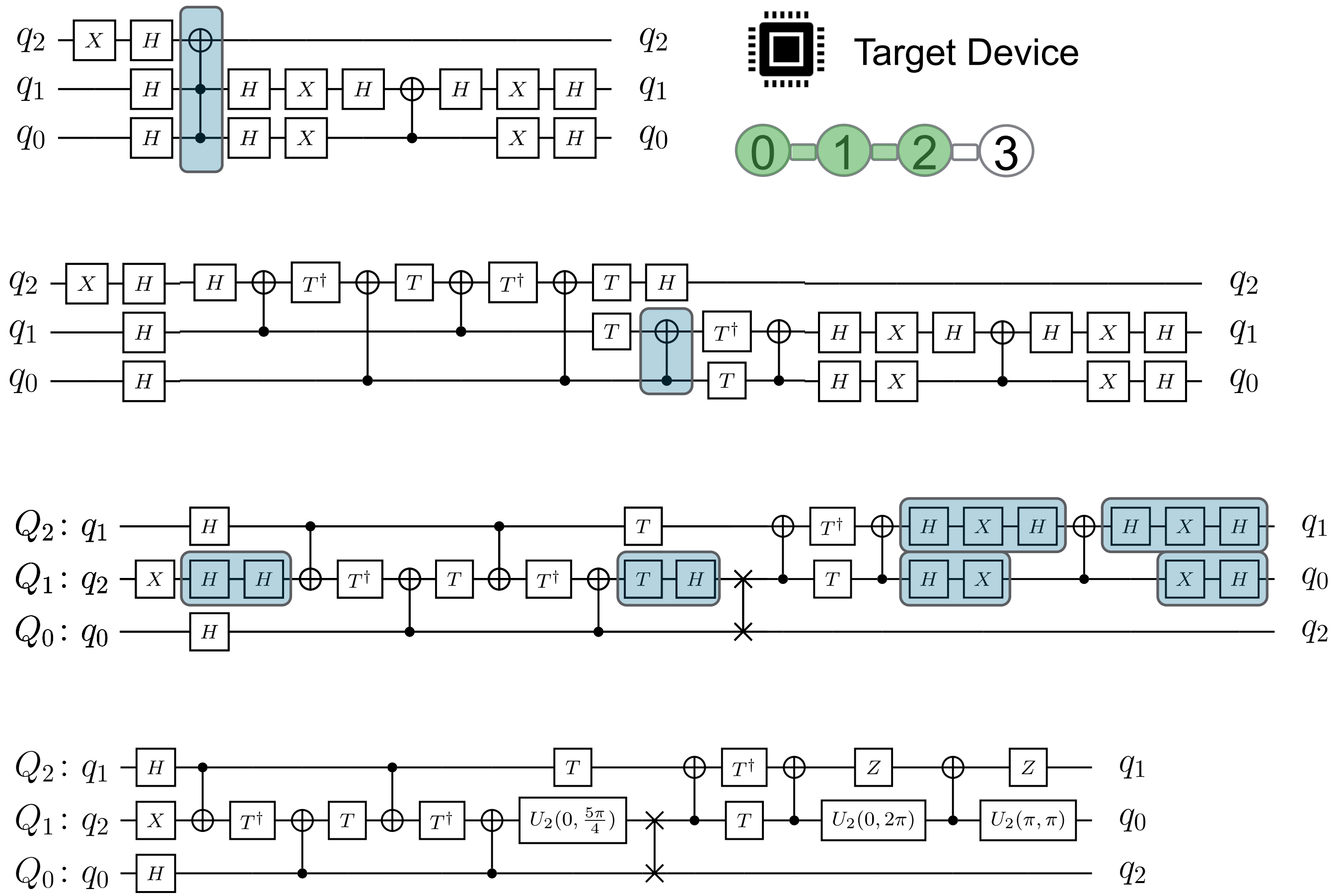
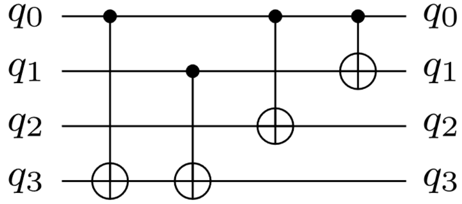
# The Quantum Circuit Compilation Flow

- Conceptional algorithm
- Limited Gate Set
- Synthesis
- **Limited Connectivity**
- **Mapping**
- Limited Fidelity and Coherence
- Optimizations
- Actual Realization

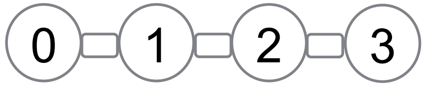


# Quantum Circuit Mapping

Decomposed Circuit

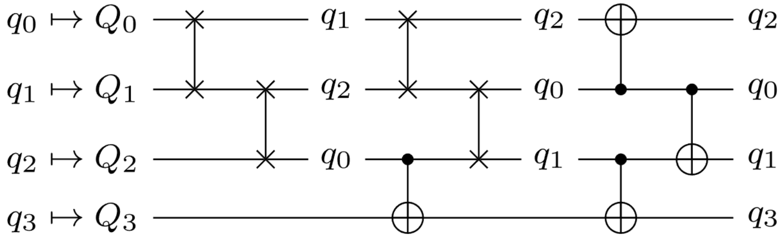



Target Device

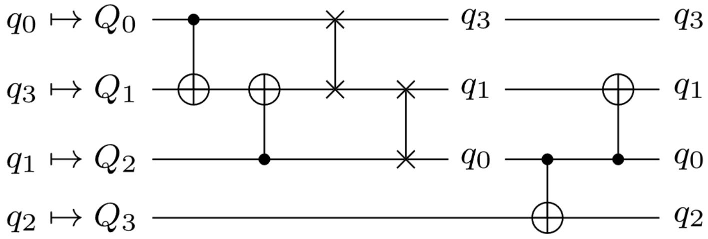



- Different approaches for mapping offer trade-off between runtime vs quality of result

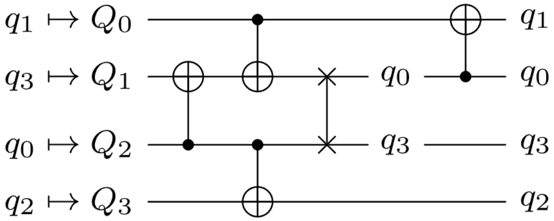
Naive Approach




Heuristic Approach

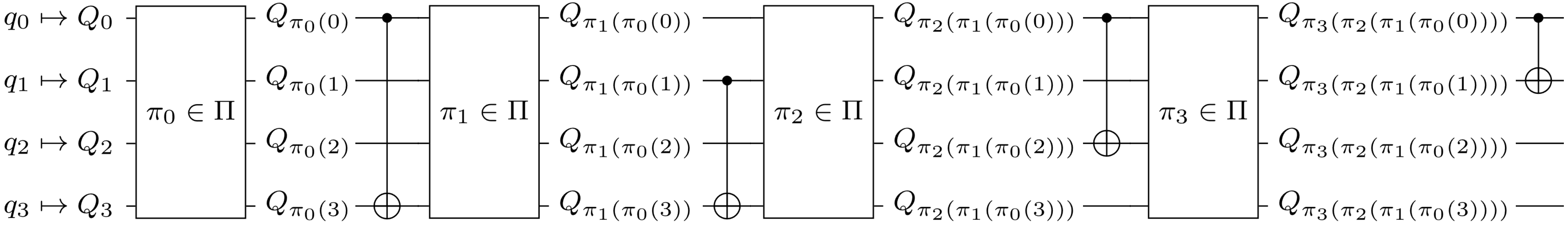



Optimal Approach

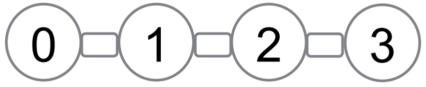


# Optimal Quantum Circuit Mapping

- Different Objectives: gate-count, depth, fidelity



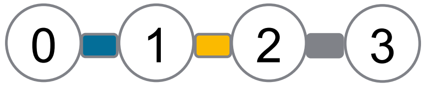

Target Device

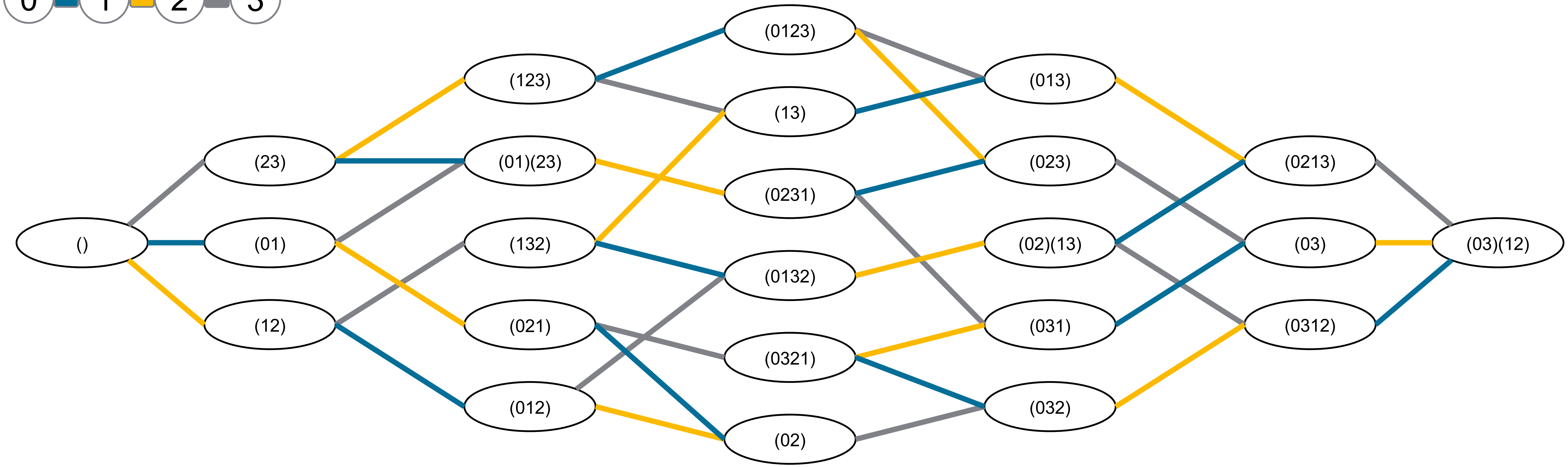



- Valid mapping makes every gate executable
- Each permutation layer consists of $n!$ possible choices
- Permutations are eventually realized as sequences of swaps
- Goal: minimize the number of swaps

# Visualizing the Search Space

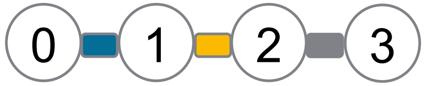
Target Device

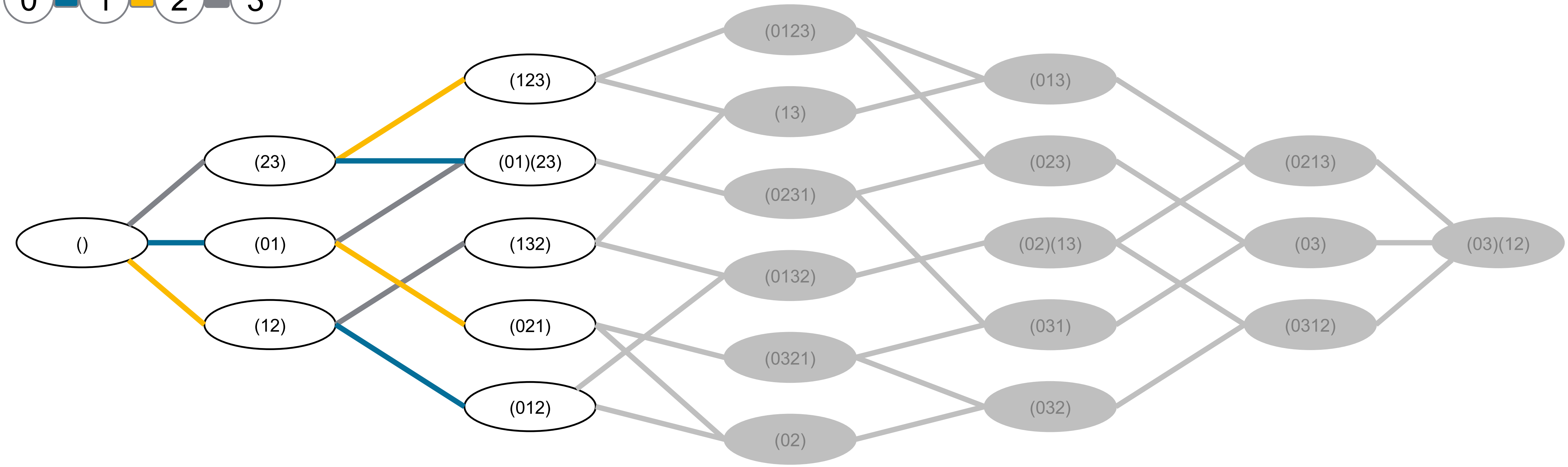


# Limiting the Search Space

Target Device

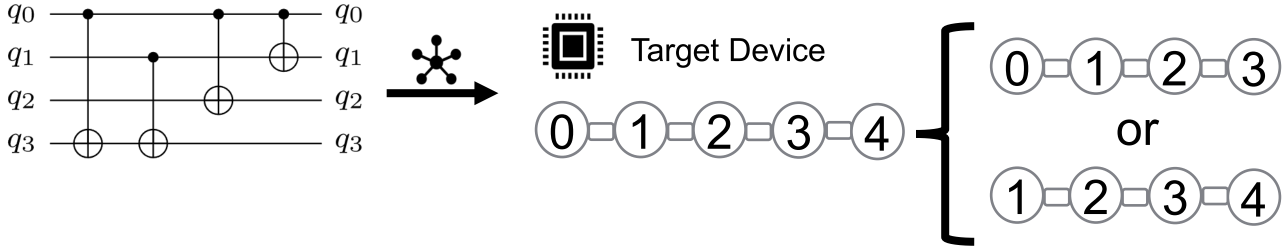



If $K$ is the longest, direct path on the architecture, then only those permutations with up to $K-1$ swaps need to be considered!

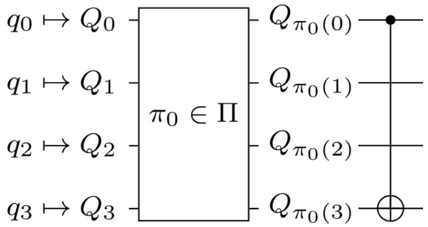
# Limiting the Search Space II

- Considering subsets of qubits




Apply search space limitation on each subset

- Permutations not affecting the qubits of a gate can be ignored



() and (12) can be ignored

# Experimental Results

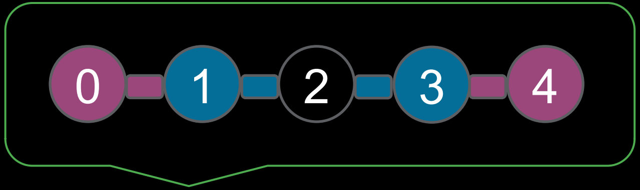
| Benchmark | | | | Without Subgraphs | | | | | With Subgraphs | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | JKQ QMAP [20] | | Architecture Limit (Section V-A) | | | JKQ QMAP [20] | | Subgraph Limit (Section V-B) | | |
| Name | $n$ | $\|G\|$ | $c$ | $\|\Pi\|$ | $t_{ref}$ [s] | $\|\Pi'\|$ | $t_{prop}$ [s] | $t_{ref}/t_{prop}$ | $\|\Pi\|$ | $t_{ref}$ [s] | $\|\Pi'\|$ | $t_{prop}$ [s] | $t_{ref}/t_{prop}$ |
| 4_49_16 | 5 | 217 | 207 | 26 040 | >1 h | 3 472 | **171.15** | - | 26 040 | >1 h | 3 472 | **171.07** | - |
| hwb4_49 | 5 | 233 | 213 | 27 960 | >1 h | 3 728 | **198.45** | - | 27 960 | >1 h | 3 728 | **199.57** | - |
| mod10_171 | 5 | 244 | 285 | 29 280 | >1 h | 3 904 | **291.04** | - | 29 280 | >1 h | 3 904 | **283.86** | - |
| mini-alu_167 | 5 | 288 | 330 | 34 560 | >1 h | 4 608 | **477.25** | - | 34 560 | >1 h | 4 608 | **475.06** | - |
| one-two-three-v0_97 | 5 | 290 | 234 | 34 800 | >1 h | 4 640 | **364.29** | - | 34 800 | >1 h | 4 640 | **363.97** | - |
| alu-v2_31 | 5 | 451 | 375 | 54 120 | >1 h | 7 216 | **1 418.29** | - | 54 120 | >1 h | 7 216 | **1 413.01** | - |
| decod24-v3_45 | 5 | 150 | 156 | 18 000 | 1 750.86 | 2 400 | **57.53** | 30.43 | 18 000 | 1 738.25 | 2 400 | **58.65** | 29.64 |
| aj-e11_165 | 5 | 151 | 129 | 18 120 | 1 691.31 | 2 416 | **47.90** | 35.31 | 18 120 | 1 689.52 | 2 416 | **47.94** | 35.25 |
| 4mod7-v1_96 | 5 | 164 | 123 | 19 680 | 1 679.76 | 2 624 | **51.87** | 32.39 | 19 680 | 1 655.91 | 2 624 | **51.49** | 32.16 |
| alu-v2_32 | 5 | 163 | 117 | 19 560 | 1 481.97 | 2 608 | **48.19** | 30.75 | 19 560 | 1 477.13 | 2 608 | **47.59** | 31.04 |
| 4gt10-v1_81 | 5 | 148 | 111 | 17 760 | 1 319.27 | 2 368 | **40.84** | 32.30 | 17 760 | 1 347.52 | 2 368 | **41.16** | 32.74 |
| one-two-three-v0_98 | 5 | 146 | 108 | 17 520 | 1 227.19 | 2 336 | **44.85** | 27.36 | 17 520 | 1 254.30 | 2 336 | **46.91** | 26.74 |
| one-two-three-v1_99 | 5 | 132 | 108 | 15 840 | 1 071.84 | 2 112 | **35.53** | 30.16 | 15 840 | 1 056.80 | 2 112 | **35.24** | 29.99 |
| 4gt5_77 | 5 | 131 | 99 | 15 720 | 914.56 | 2 096 | **31.16** | 29.35 | 15 720 | 927.59 | 2 096 | **31.53** | 29.42 |
| 4gt13_91 | 5 | 103 | 93 | 12 360 | 632.21 | 1 648 | **20.17** | 31.34 | 12 360 | 651.83 | 1 648 | **20.27** | 32.15 |
| miller_11 | 3 | 50 | 27 | 6 000 | 624.63 | 800 | **0.33** | 1 880.11 | 300 | 0.11 | 150 | **0.06** | 1.91 |
| alu-v4_36 | 5 | 115 | 87 | 13 800 | 594.61 | 1 840 | **20.36** | 29.20 | 13 800 | 586.32 | 1 840 | **20.70** | 28.33 |
| 4gt5_76 | 5 | 91 | 84 | 10 920 | 444.35 | 1 456 | **0.35** | 1 254.61 | 10 920 | 458.90 | 1 456 | **0.35** | 1 298.82 |
| decod24-v1_41 | 5 | 85 | 84 | 10 200 | 344.17 | 1 360 | **11.10** | 31.00 | 10 200 | 341.90 | 1 360 | **11.46** | 29.84 |
| decod24-v2_43 | 4 | 52 | 27 | 6 240 | 254.36 | 832 | **0.24** | 1 047.78 | 1 248 | 26.65 | 468 | **0.32** | 83.75 |
| 4mod5-v1_23 | 5 | 69 | 66 | 8 280 | 198.41 | 1 104 | **6.55** | 30.31 | 8 280 | 210.68 | 1 104 | **6.56** | 32.11 |
| 4gt13_92 | 5 | 66 | 78 | 7 920 | 193.88 | 1 056 | **5.79** | 33.49 | 7 920 | 196.83 | 1 056 | **5.75** | 34.23 |
| rd32_270 | 5 | 84 | 54 | 10 080 | 155.96 | 1 344 | **5.59** | 27.89 | 10 080 | 155.42 | 1 344 | **5.64** | 27.58 |
| 4mod5-v0_18 | 5 | 69 | 48 | 8 280 | 135.65 | 1 104 | **3.52** | 38.57 | 8 280 | 144.92 | 1 104 | **3.44** | 42.15 |
| one-two-three-v2_100 | 5 | 69 | 48 | 8 280 | 133.85 | 1 104 | **0.23** | 577.65 | 8 280 | 131.56 | 1 104 | **0.23** | 580.92 |
| mod5d2_64 | 5 | 53 | 42 | 6 360 | 84.36 | 848 | **0.22** | 385.96 | 6 360 | 83.82 | 848 | **0.22** | 381.36 |
| alu-v1_28 | 5 | 37 | 30 | 4 440 | 65.02 | 592 | **0.13** | 506.02 | 4 440 | 65.82 | 592 | **0.13** | 508.11 |
| 3_17_13 | 3 | 36 | 18 | 4 320 | 35.31 | 576 | **0.26** | 137.53 | 216 | 0.08 | 108 | **0.04** | 1.92 |
| rd32-v1_68 | 4 | 36 | 18 | 4 320 | 12.85 | 576 | **0.13** | 102.67 | 864 | 0.51 | 324 | **0.17** | 2.96 |
| rd32-v0_66 | 4 | 34 | 18 | 4 080 | 12.83 | 544 | **0.12** | 103.10 | 816 | 0.51 | 306 | **0.17** | 2.97 |
| 4gt13_90 | 5 | 107 | 114 | 12 840 | 10.73 | 1 712 | **0.30** | 35.36 | 12 840 | 10.90 | 1 712 | **0.30** | 36.02 |
| mod5mils_65 | 5 | 35 | 24 | 4 200 | 2.53 | 560 | **0.12** | 21.51 | 4 200 | 2.49 | 560 | **0.12** | 21.13 |
| alu-v4_37 | 5 | 37 | 30 | 4 440 | 2.26 | 592 | **0.15** | 14.71 | 4 440 | 2.25 | 592 | **0.16** | 14.47 |
| one-two-three-v3_101 | 5 | 70 | 66 | 8 400 | 1.69 | 1 120 | **0.26** | 6.45 | 8 400 | 1.65 | 1 120 | **0.27** | 6.20 |
| alu-v3_34 | 5 | 52 | 51 | 6 240 | 1.57 | 832 | **0.19** | 8.44 | 6 240 | 1.60 | 832 | **0.19** | 8.60 |
| decod24-v0_38 | 4 | 51 | 27 | 6 120 | 1.33 | 816 | **0.31** | 4.26 | 1 224 | 35.77 | 459 | **0.82** | 43.49 |
| qe_qft_5 | 5 | 107 | 9 | 12 840 | 0.99 | 1 712 | **0.22** | 4.54 | 12 840 | 0.97 | 1 712 | **0.22** | 4.41 |
| 4mod5-v0_19 | 5 | 35 | 30 | 4 200 | 0.96 | 560 | **0.83** | 1.16 | 4 200 | 1.43 | 560 | **0.82** | 1.74 |
| 4gt11_82 | 5 | 27 | 45 | 3 240 | 0.93 | 432 | **0.24** | 3.92 | 3 240 | 1.37 | 432 | **0.24** | 5.79 |
| alu-v3_35 | 5 | 37 | 30 | 4 440 | 0.87 | 592 | **0.15** | 5.63 | 4 440 | 1.07 | 592 | **0.15** | 6.93 |

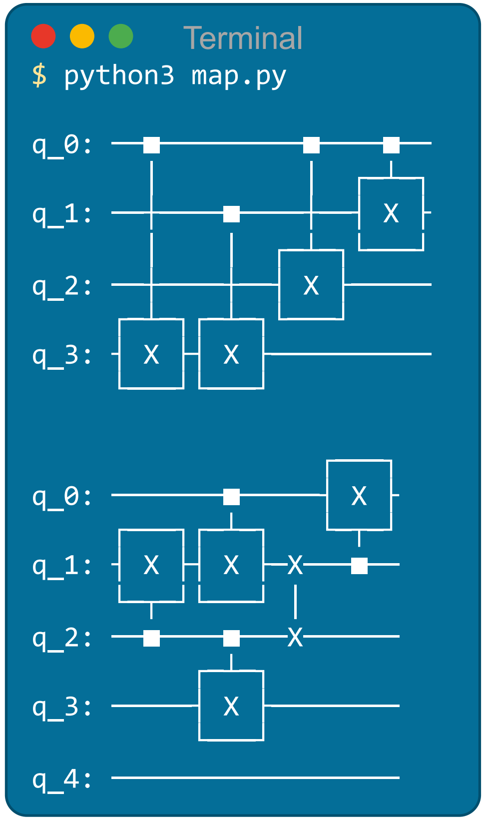
# JKQ QMAP

**https://github.com/iic-jku/qmap** or **pip install jkq.qmap**

map.py

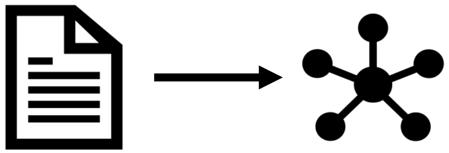
```
from qiskit import *
from jkq import qmap

q = QuantumRegister(4, 'q')
circ = QuantumCircuit(q)
circ.cx(q[0], q[3])
circ.cx(q[1], q[3])
circ.cx(q[0], q[2])
circ.cx(q[0], q[1])
print(circ.draw())

results = qmap.compile(circ, arch='IBMQ_Bogota', method='exact')

m_circ = QuantumCircuit.from_qasm_str(result.mapped_circuit)
print(m_circ.draw())
```



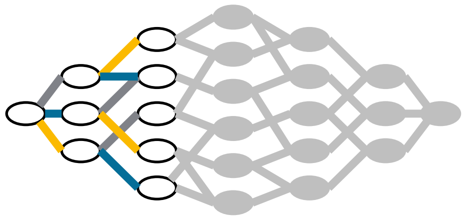



# Conclusion

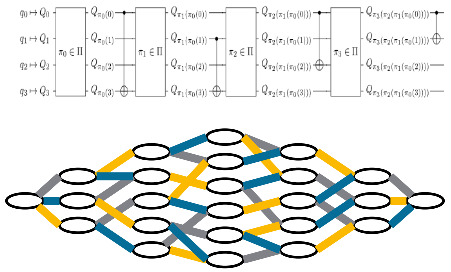
Quantum Circuit Mapping



Limiting the Search Space



Optimal Solution



QMAP